



FIG. 11. Percent permanent bore enlargement for 100 per cent overstrain vs. diameter ratio.

The elastic recovery at the bore is given by the Lamé equations and Eq. (4).

$$\epsilon_{tae} = \frac{\sigma_y}{E} \frac{1.08 \ln W}{W^2 - 1} [(1 + \mu) W^2 + (1 - \mu)] \quad (27)$$

Subtracting Eq. (27) from Eq. (26) yields:

$$\epsilon_{taperm} = [\mu + (2 - \mu)W^2] \left[ \frac{1}{2} - \frac{1.08 \ln W}{W^2 - 1} \right] \quad (28)$$

This equation is plotted in Fig. 11 for  $\mu = 0.3$  and is in good agreement with the experimental values.

## SUMMARY

The application of the autofrettage principle to materials of 165,000 psi yield strength has been experimentally investigated. Resulting from this investigation are a series of relationships for the stresses and displacements in overstrained thick-wall cylinders in the open-end condition. These relationships, which can be used for pressure vessel design purposes, are based on the von Mises yield criterion but incorporating empirical constants for simplicity. The agreement between these relationships and the experimental data is good.

## REFERENCES

1. WEIGLE, R. E. "Elastic-Plastic Analysis of a Cylindrical Tube" Watervliet Arsenal Technical Report WVTRR-6007, March 1960.
2. FAUPEL, J. H. "Residual Stresses in Heavy Wall Cylinders", *J. Franklin Inst.* pp. 409-419, May 1955.
3. FAUPEL, J. H. and FURBECK, A. R. "Influence of Residual Stress on Behavior of Thick-Wall Closed-End Cylinders" *Trans. A.S.M.E.* 75, No. 3, April 1953.
4. KOITER, W. T. "On Partially Plastic Thick-Walled Tubes" C. B. Biezeno Anniversary Volume on Applied Mechanics, N. V. de Technische Uitgeverij, H., Stam. Haarlem, 1953.
5. LANGENBERG, F. C. "Effect of Cold Working on the Strength of Hollow Cylinders" *Trans. Amer. Soc. Steel Treating*, 8, No. 4, October 1925.
6. MAC GREGOR, C. W., COFFIN, L. F., Jr., and FISHER, J. C. "Partially Plastic Thick-Walled Tubes" *J. Franklin Inst.* pp. 135-158, February, 1948.
7. NEWHALL, D. H. "Selected Design Data Pertaining to Gun Tubes and High-Pressure Vessels" Watertown Arsenal Report No. WGD-4, December 1943.
8. NEWHALL, D. H. "Plastic Strains in Thick Hollow Cylinders Overstrained by Internal Pressure" Watertown Arsenal Report No. WGD-7, January 1944.
9. PRAGER, W. "Stress-Strain Laws of the Mathematical Theory of Plasticity Discussion" *J. of Appl. Mech.* 70, pp. 226-233, September 1948.
10. SOPWITH, D. G. and DE G. ALLEN, D. N. "The Stresses and Strains in a Partially Plastic Thick Tube under Internal Pressure and End Load", *Proc. Roy. Soc., London*. Ser. A. 205, pp. 69-83, 1951.
11. STEEL, M. C. and YOUNG, JOHN "An Experimental Investigation of Over-Straining in Mild-Steel Thick-Walled Cylinders by Internal Fluid Pressure" *Trans. A.S.M.E.* pp. 355-363, April 1952.
12. TIMOSHENKO, S. *Strength of Materials, Part II*. Van Nostrand, 1942.
13. TURNER, L. B. "The Stresses in a Thick Hollow-Cylinder Subjected to Internal Pressure" *Trans. Camb. Phil. Soc.* XXI, No. XIV. pp. 377-396.
14. WARREN, A. G. "Autofrettage", *Symposium on Internal Stresses in Metals and Alloys*, pp. 209-218, Institute of Metals, 1947.